CRUNCH Seminars at Brown, Division of Applied Mathematics

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Deep neural network surrogates for non-smooth quantities of interest in shape uncertainty quantification

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We consider the point evaluation of the solution to interface problems with geometric uncertainties, where the uncertainty in the obstacle is described by a high-dimensional parameter y ∈ [−1, 1]^d, d ∈ N. We focus in particular on an elliptic interface problem and a Helmholtz transmission problem. Point values of the solution in the physical domain depend in general non-smoothly on the high-dimensional parameter, posing a challenge when one is interested in building surrogates. Indeed, high-order methods show poor convergence rates, while methods which are able to track discontinuities usually suffer from the so-called curse of dimensionality. For this reason, in this work we propose to build surrogates for point evaluation using deep neural networks. We provide a theoretical justification for why we expect neural networks to provide good surrogates. Furthermore, we present extensive numerical experiments showing their good performance in practice. We observe in particular that neural networks do not suffer from the curse of dimensionality, and we study the dependence of the error on the number of point evaluations (that is, the number of discontinuities in the parameter space), as well as on several modeling parameters, such as the contrast between the two materials and, for the Helmholtz transmission problem, the wavenumber.