CRUNCH Seminars at Brown, Division of Applied Mathematics

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Paper Review: Predicting the solutions of heterogeneous elliptic PDEs with a confidence interval by probabilistic convolutional neural networks

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In this work we introduce a framework for constructing lightweight numerical solvers for partial di erential equations (PDEs) using convolutional neural networks. These solvers are able to e ectively reduce the computational demands of traditional numerical methods into a single forward-pass of a convolutional network. The network architecture is designed to predict pointwise Gaussian posterior distributions, with weights trained to minimize the associated negative log likelihood of the observed solutions. This setup facilitates simultaneous training and uncertainty quantification for the network's solutions, allowing the solver to provide pointwise predictive uncertainties for its predictions. The performance of the framework is demonstrated on three distinct classes of PDEs consisting of two linear elliptic problem setups and a nonlinear Poisson problem. After a single offline training procedure for each class, the network is capable of accurately approximating solutions 30 times faster than traditional numerical methods. Additionally, an analysis of the predicted pointwise uncertainties is presented with experimental evidence establishing the validity of the network's uncertainty quantification schema.