CRUNCH Seminars at Brown, Division of Applied Mathematics

Friday - May 14, 2021

PExact imposition of boundary conditions with distance functions in physics-informed deep neural networks

N. Sukumara/Ankit Srivastava, Illinois Institute of Technology

In this paper, we introduce a new approach based on distance fields to exactly impose boundary conditions in physics-informed deep neural networks. The challenges in satisfying Dirichlet boundary conditions in mesh-free and particle methods are well-known. This issue is also pertinent in the development of physics informed neural networks (PINN)for the solution of partial differential equations. We introduce geometry-aware trial functions in artificial neural networks to improve the training in deep learning for partial differential equations. To this end, we use concepts from constructive solid geometry (R-functions) and generalized barycentric coordinates (mean value potential fields) toconstructφ(x), an approximate distance function to the boundary of a domain in. To exactly impose homogeneous Dirichlet boundary conditions, the trial function is taken asφ(x) multiplied by the PINN approximation, and its generalization via transfinite interpolation is used to a priori satisfy inhomogeneous Dirichlet (essential), Neumann(natural), and Robin boundary conditions on complex geometries. In doing so, we eliminate modeling error associated with the satisfaction of boundary conditions in a collocation method and ensure that kinematic admissibility is met pointwise in a Ritz method. With this new ansatz, the training for the neural network is simplified: sole contribution to the loss function is from the residual error at interior collocation points where the governing equation is required to be satisfied. Numerical solutions are computed using strong form collocation and Ritz minimization. To convey the main ideas and to assess the accuracy of the approach, we present numerical solutions for linear and nonlinear boundary-value problems over convex and nonconvex polygonal domains as well as over domains with curved boundaries. Benchmark problems in one dimension for linear elasticity, advection-diffusion, and beam bending; and in two dimensions for the steady-state heat equation, Laplace equation, biharmonic equation (Kirchhoffplate bending), and the nonlinear Eikonal equation are considered. The construction of approximate distance functions using R-functions extends to higher dimensions, and we showcase its use by solving a Poisson problem with homogeneous Dirichlet boundary conditions over the four-dimensional hypercube. The proposed approach consistently outperforms a standard PINN-based collocation method, which underscores the importance of exactly (a priori) satisfying the boundary condition when constructing a loss function in PINN. This study provides a pathway for mesh-free analysis to be conducted on the exact geometry without domain discretization.