Deep neural networks, often required in complex learning tasks such as image classification, are hard to train. We propose a novel formulation, inspired by the graph kernel network (GKN), that allows for deep layers. Our nonlocal kernel network (NKN) stems from the interpretation of the neural network as a discrete nonlocal diffusion reaction problem that, in the limit of infinite layers, is equivalent to a parabolic nonlocal equation, whose stability is analyzed via nonlocal vector calculus. The resemblance with graph neural networks allows NKNs to capture long-range dependencies in the feature space, while the continuous treatment of node-to-node interactions makes NKNs resolution independent. Furthermore, the resemblance with neural ODEs, reinterpreted in a nonlocal sense, and the stable network dynamics between layers allow for generalization of NKN's optimal parameters from shallow to deep networks. This fact enables the use of shallow-to-deep initialization techniques. Our tests show that NKNs outperform baseline methods in both PDE learning tasks (such as learning Poisson and multi-dimensional Darcy's equations) and image classification tasks and generalize well to different resolutions and depths.