First-Derivatives

Gauss-Lobatto

\[ D_{ij} = \begin{cases} \frac{\alpha-N(N+2\alpha+1)}{2(\alpha+2)} & i = j = 0 \\ \frac{(-1)^{N+1}}{N!(\alpha+1)\Gamma(2\alpha+1)} \frac{1}{(1+z_j)P^{(\alpha)}_N(z_j)} & i = 0, j \in [1, N-1] \\ \frac{(-1)^N}{\Gamma(N+2\alpha+1)} P^{(\alpha)}_N(z_i) & i = 0, j = N \\ \frac{(-1)^N N!}{\Gamma(N+2\alpha+1)} \frac{1}{1+z_i} & i \in [1, N-1], j = 0 \\ \frac{\Gamma(N+2\alpha+1)}{\Gamma(N+\alpha+1)\Gamma(2\alpha+1)} \frac{1}{(1-z_j)P^{(\alpha)}_N(z_j)} & i \neq j, i, j \in [1, N-1] \\ \frac{1}{z_i-z_j} P^{(\alpha)}_N(z_j) & i = j \in [1, N-1] \\ \frac{\Gamma(\alpha+1)}{\Gamma(N+2\alpha+1)} \frac{1}{1-z_i} & i = N, j = 0 \\ \frac{\Gamma(N+2\alpha+1)}{\Gamma(N+\alpha+1)\Gamma(2\alpha+1)} \frac{1}{(1-z_j)P^{(\alpha)}_N(z_j)} & i = N, j \in [1, N-1] \\ \frac{\alpha+N(N+2\alpha+1)}{2(\alpha+2)} & i = j = N \end{cases} \]

Gauss

\[ \tilde{D}_{ij} = \begin{cases} \frac{(\alpha+1)z_i}{1-(z_j)^2} & i = j \\ \frac{P^{(\alpha)}_{N+1}(z_i)}{(z_i-z_j)(P^{(\alpha)}_{N+1})(z_i)} & i \neq j \end{cases} \]
The Gauss collocation points, $z_i$, for $N = 12$ for the ultraspherical polynomial, $P_{12}^{(a)}(x)$, as a function of $\alpha$.

The Ultraspherical Gauss-Lobatto collocation points, $x_i$, for $N = 12$ for the polynomial, $P_{12}^{(a)}(x)$, as a function of $\alpha$. 