Modeling and optimization of colloidal micro-pumps

To cite this article: D Liu et al 2004 J. Micromech. Microeng. 14 567

View the article online for updates and enhancements.

Related content
- Simulations of dynamic self-assembly of paramagnetic microspheres
  D Liu, M R Maxey and G E Karniadakis
- Direct numerical simulation of horizontal open channel flow with finite-size, heavy particles at low solid volume fraction
  Aman G Kidanemariam, Clemens Chan-Braun, Todor Doychev et al.
- Emergent behavior in active colloids
  Andreas Zöttl and Holger Stark

Recent citations
- Numerical Characterisation of Active Drag and Lift Control for a Circular Cylinder in Cross-Flow
  Philip McDonald and Tim Persoons
- Simulation of flow around rigid vegetation stems with a fast method of high accuracy
  Don Liu et al
- Modal Spectral Element Solutions to Incompressible Flows over Particles of Complex Shape
  Don Liu and Yonglai Zheng

IOP ebooks

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.
Modeling and optimization of colloidal micro-pumps

D Liu, M Maxey and G E Karniadakis

Division of Applied Mathematics, Brown University, Providence, RI 02912, USA
E-mail: gk@dam.brown.edu

Received 3 November 2003
Published 19 January 2004
Online at stacks.iop.org/JMM/14/567 (DOI: 10.1088/0960-1317/14/4/018)

Abstract
Manipulating micro-particles is very important in microfluidic applications, such as biomedical flows and self-assembled structures. Here, flows generated by the forced motion of colloidal micro-particles in a microchannel are investigated. The force coupling method combined with the spectral/hp element method is used to numerically simulate the dynamics of the flow, while a penalty method is used to determine the required forces on the particles. The pumping motion is investigated for two specific systems: a peristaltic micro-pump and a gear micro-pump. We verify the accuracy of the simulations and then for each system, we investigate the net flow rate as a function of pump frequency and channel dimension, and present optimization results. The results for the net flow rate are comparable to and within the range of the experimental data.

1. Introduction
This work was motivated by the experiments of Terray et al [1], who have designed and operated colloidal micro-devices (pumps and valves) for microfluidic control. Pump designs that employ inertial, centrifugal action, e.g. impeller-based pumps, are inappropriate for microfluidic applications due to the very small Reynolds number. However, designs based on positive-displacement methods can be quite effective. To this end, Terray et al [1] have developed two positive-displacement micro-pumps by using colloidal microspheres as the active flow-control element. Small microspheres, at colloidal scales, can be manipulated remotely using electric or magnetic fields or optical trapping; the latter was employed by Terray et al in their experiments. Remote activation techniques are particularly attractive in controlling micro-devices as there is no need to interface directly the microfluidic application with the macroscopic environment—a major drawback in many recent designs of micro-devices.

In these devices, gravitational settling of the microspheres is negligible and the effect of Brownian motion is small compared to the imposed control forces. The number of colloidal microspheres used is also small. In the first micro-pump of Terray et al there are six particles forming a chain along the channel. In the second micro-pump there are four particles, separated in two pairs that counter-rotate with respect to each other. The chain of six microspheres is activated by optical trapping that induces a smooth traveling wave motion. In the second design the motion of the ‘meshing gears’ of the micro-pump is not as smooth. Both designs, however, were shown to operate successfully and induce about the same net flow rate, of the order of 1 nl hr⁻¹ for a rotation of a few Hz.

The objective of the current work is first to determine how accurate the modeling of such devices is, and second to use simulation to improve and possibly optimize their performance. Particulate microflows are difficult to model accurately [2] and modeling assumptions cannot be verified directly. However, recent work based on atomistic simulations in [3] has shown that the continuum hypothesis may be valid even for particles as small as 2 nm. To this end, we use continuum-based methods to model the two aforementioned micro-pumps at various conditions. In particular, we employ the force coupling method that has been validated extensively for microflows in [4, 5].

In the following, we first present some details of the mathematical modeling and subsequently study the peristaltic micro-pump and the two-lobe gear micro-pump. In particular, we investigate their performance for a wide range of rotational speeds and also with respect to the width of the channel. We then conclude with a summary and a discussion on the limitations of these micro-pump designs as well as of the modeling approach. Finally we have some direct numerical simulation (DNS) validations in an appendix.
2. Mathematical modeling

The particles that we consider in this work are smaller than 3 µm but larger than about 100 nm. In this size range we can simulate particulate microflows as a continuum with possible corrections due to partial slip at the wall or due to random effects. The robustness of continuum calculations in this context has been demonstrated previously in [6, 7] for spheres approaching a plane wall. More recently, a systematic molecular dynamics (MD) study was undertaken by Drazer et al. [3] for a colloidal spherical particle passing through a nanotube containing a partially wetting fluid. For a generalized Lennard–Jones liquid, it was demonstrated that the MD simulations are in good agreement with the continuum simulations of [8] despite the large thermal fluctuations present in the system. This is true even for very small size particles of the order of 2 nm, i.e. much smaller than the ones we consider here.

2.1. Force coupling method

We therefore adopt a continuum description of the flow and use the force coupling method (FCM) [9, 10] to represent the dynamics of the particles in the flow. The governing equations of fluid motion for the fluid velocity \( \mathbf{u}(x, t) \) are given by

\[
\rho \frac{D \mathbf{u}}{D t} = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}(x, t) \tag{1}
\]

\[
\nabla \cdot \mathbf{u} = 0 \tag{2}
\]

where \( \rho \), \( p \) and \( \mu \) are the fluid density, pressure and viscosity, respectively. The source term \( \mathbf{f}(x, t) \) represents the sum of two-way coupling forces from each spherical particle \( n \) centered at \( Y^{(n)}(t) \). For a single particle we have

\[
f_i(x, t) = F_i \Delta \sigma_1(x, x - Y(t)) + G_{ij} \frac{\partial \Delta \sigma_2(x, x - Y(t))}{\partial x_j} \tag{3}
\]

where both \( \Delta \sigma_1(x, x) \) and \( \Delta \sigma_2(x, x) \) are Gaussian distribution functions of the form

\[
\Delta \sigma_1(x, x) = \frac{(2\pi \sigma_1^2)^{-1}}{\sqrt{\pi}} \exp\left(-x^2/(2\sigma_1^2)\right). \tag{4}
\]

The first term in equation (3) is a finite force monopole of strength \( F \) while the second term is a force dipole of strength \( G_{ij} \). The monopole strength is set by the sum of the external forces, due to optical trapping, magnetic field or gravity, acting on the particle as well as the inertia of the particle. If \( m_p \) is the mass of the particle and \( m_F \) is the mass of the displaced fluid then

\[
\mathbf{F} = F^{(1)} - (m_p - m_F) \frac{d\mathbf{V}}{dt}. \tag{5}
\]

Additionally, contact forces between particles or a particle and a wall may be included in determining \( \mathbf{F} \).

The second term in (3) is the force dipole, which has symmetric and antisymmetric components. The symmetric, streslet component of the force dipole \( G_{ij} \) is chosen so that

\[
\int \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \Delta \sigma_2(x, x - Y) \, dx = 0. \tag{6}
\]

This is set through a dipole iteration scheme to minimize the integral-averaged strain rate inside the volume occupied by particles, down to 1% of the ratio of the particle velocity to its radius, see [4]. The antisymmetric components of the force dipole correspond to external torques acting on the particle. The values of the length scales \( \sigma_1 \) and \( \sigma_2 \) in (3) are directly related to the particle radius, with \( \sigma_1/\sigma_2 = (\sqrt{3}/\pi)^{1/3} \). These values ensure that in viscous-dominated flows the correct Stokes drag and torque on a single isolated particle are achieved (details are given in [9, 10]).

The velocity of each particle \( \mathbf{V}(t) \) is evaluated from a volume integral of the local fluid velocity with the monopole Gaussian distribution function as

\[
\mathbf{V}(t) = \int_D \mathbf{u}(x, t) \Delta \sigma_1(x, x - Y(t)) \, dx. \tag{7}
\]

The angular velocity \( \Omega \) of the particle is computed from a local volume average of the vorticity \( \mathbf{W}(x, t) \) with the dipole Gaussian distribution function as

\[
\Omega = \frac{1}{2} \int_D \mathbf{W}(x, t) \Delta \sigma_2(x, x - Y(t)) \, dx. \tag{8}
\]

The new position of each particle is calculated from

\[
\mathbf{V}(t) = \frac{d\mathbf{V}(t)}{dt}. \tag{9}
\]

A spectral/hp element method has been used to solve for the primitive variables \( \mathbf{u}, p \) in the Navier–Stokes equations and further details can be found in [11]. In these micro-devices the flows are dominated by viscous forces and a Reynolds number based on the diameter of a particle and the particle velocity is of the order of \( O(10^{-4}) \). It is appropriate then to neglect the fluid inertia in (1) and to solve the corresponding equation for Stokes flow

\[
0 = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}(x, t). \tag{10}
\]

Similarly, we can neglect the influence of particle inertia in (5) and the external force required for each particle is equal to the monopole force \( F \).

2.2. Penalty method

The standard FCM scheme is based on a mobility formulation, where the force (5) acting on the particle is prescribed and the velocity is determined from the computed flow (7). In the pump micro-devices considered here the motion of the particles and their velocities are prescribed, and finding the forces and matching the motion are our goals. But the actual forces from the optical trapping acting on the particles are unknown. In order to solve this resistance problem within the FCM scheme, we incorporate a penalty method.

For example, in order to find the force needed for the particle numbered by \( n \) to generate the desired motion, we formulate a time-dependent model for the force \( F^{(n)}(t) \) and the particle velocity difference between the target value \( \mathbf{V}^{(n)}(t) \) and the computed value \( \mathbf{V}^{(n)}(t) \) as

\[
\frac{dF^{(n)}(t)}{dt} = \lambda \left[ V^{(n)}_1(t) - \lambda^{(n)}(t) \right] \tag{11}
\]

where \( \lambda \) is the penalty parameter. The magnitude of \( \lambda \) controls the rate of convergence; the larger its value the tighter the control of the particle motion and the faster the convergence. However, for stability reasons, \( \lambda \) cannot exceed a certain level. Upon convergence, the difference \( \left| V^{(n)}_1(t) - \lambda^{(n)}(t) \right| \) is basically zero and the force \( F^{(n)}(t) \) becomes independent of time \( t \), and the initially unknown value of the force is obtained. This procedure may be applied to both steady and unsteady flows.
no-slip boundary conditions were specified on the four rigid V particles remains small but the particles do not overlap. Using figure 1. The length of the microchannel is 24 $\mu$m in the $x_1$ direction, the height is 6 $\mu$m in the $x_2$ direction and the initial width is 4 $\mu$m in the $x_3$ direction. In the simulations we will vary the width of the channel to investigate its effect on the net flow rate. Six particles with the diameter $2a = 3$ $\mu$m were initially assembled as a sine-wave chain along the centerline of the microchannel. Periodic boundary conditions for the flow were applied at the channel ends at $x_1 = 0$ and 24 $\mu$m while no-slip boundary conditions were specified on the four rigid sidewalls of the channel in the $x_2$ and $x_3$ directions.

In order to produce a pumping effect, a transverse traveling wave motion of the following form is enforced for the $n$th microsphere ($n = 0, 1, \ldots, 5$):

$$Y_n^{(0)}(t) = A_2 \sin \left[ \omega t + \left( n - \frac{1}{2} \right) \frac{\pi}{2} \right]$$ (12)

$$Y_n^{(n)}(t) = Y_1^{(n)}(0) + (-1)^n A_1 \sin[2\omega t]$$ (13)

where $A_2 = 1.5$ $\mu$m is the amplitude of the traveling wave motion in the transverse $x_2$ direction, $A_1 = 0.2$ $\mu$m is the amplitude of the oscillatory motion along the $x_1$ direction and $\omega$ is the frequency. The amplitude of the transverse oscillations is equal to the particle radius $a$. The initial offsets $Y_1^{(n)}(0)$ of the particles are 7, 9.7, 12.4, 15.1, 17.8 and 20.5 $\mu$m respectively. The motion in the $x_1$ direction ensures that the gap between particles remains small but the particles do not overlap. Using equations (12) and (13) we can obtain the prescribed values of the target velocity $V_1(t)$ for each particle.

To achieve this traveling wave motion, a set of forces $F^{(n)}$ for each particle, acting in both the $x_1$ and $x_2$ directions, are computed via the aforementioned penalty method using equation (11), where $V^{(n)}(t)$ is the velocity of the microsphere $n$ computed from equation (7). The resulting forces are then the forces that, for example, the optical trapping in the experiments of Terray et al exerts on the microspheres. The penalty coefficient $\lambda$, which controls the speed of convergence, is in the range of 500–1000 [12] for oscillation frequencies in the range of 1 Hz. No external torques are applied to the particles, and the antisymmetric components of the force dipole $G_{ij}^{(n)}$ are zero. The particles are free to rotate in response to the local flow (8).

In this implementation of the FCM scheme for Stokes flow (10), an iterative procedure is used simultaneously for the computation of the flow and the forces (11). At each phase of the oscillation cycle, the particle positions (12) and (13) and the target velocities $V_1(t)$ are specified. The flow and forces are initially set from the previous phase point of the oscillation cycle. The forces are then updated from (11) and the flow is recalculated from (10). We continue this procedure until the difference in the velocity of each microsphere between successive iterations is less than 1%. Convergence is typically achieved after five iterations, see [12]. Figure 2 shows a comparison between the target and the computed velocity components of the first microsphere through several oscillation cycles.

The results for the particle velocities here are given in a non-dimensional form by using scales based on the particle radius $a$ and the angular frequency of the oscillations $\omega$. This is consistent with the prescribed motion of the particles. The time variation is scaled by $0.5T$, where $T$ is the period of the wave motion. The motion along the length of the channel has the period $0.5T$, as specified by (13). Details of the method, resolution and accuracy verification are given in the appendix.

Figure 3 shows the time variation of the volume flow rate. After a very short initial transient, the flow rate reaches a periodic state. The flow rate $Q(t)$ fluctuates about an average negative value with a period $T/2$ equal to half the period $T$ of the underlying peristaltic wave motion. The average value of the non-dimensional flow rate $Q/a^3\omega$ is $-1.426$. Terray et al [1] observed in their experiments a linear relationship between the flow rate and the frequency of the peristaltic wave, in agreement with the present results and scalings. We note that there is also a quantitative agreement with the results of Terray et al [1] in the range of 0.5 Hz to 3.5 Hz tested in the experiments. This is illustrated in figure 4 where the average flow rate is given in dimensional form against frequency. We note that in [1] the quoted maximum flow rate of 1 nl hr$^{-1}$ is erroneous, given that the maximum velocity of the tracer particle is about 3.5 $\mu$m s$^{-1}$ and the cross-section of the channel is 6 $\mu$m x 3.5 $\mu$m. A revised maximum flow rate of about 0.25 nl hr$^{-1}$ is more appropriate.

Figures 5 and 6 present the computed forces in the $x_1$ and $x_2$ directions, normalized by $6\eta a\mu o a^2$. These forces are time-periodic and switch direction within each period. The $x_2$ force component is symmetric and much larger than the $x_1$ component. The force on the leading microsphere is slightly larger than the forces on the other five. The $x_1$ components of forces are not symmetric; forces on the leading and trailing particles are mostly positive (i.e., against the net flow direction) while forces on the intermediate particles are mostly negative, i.e., toward the direction of the traveling wave. This implies that the particles in the middle of the chain contribute mainly to creating the net flow.

Next, we optimize the net flow rate with respect to the width of the micro-pump in the $x_3$ direction. We vary the width from 4 $\mu$m to 3.2, 10 and 20 $\mu$m. In figure 7 we plot the corresponding normalized forces required along with the net flow rate, versus the channel width. As the width increases,
both force components decrease, however the net flow rate first increases from 3.2 to 4.0 $\mu$m and subsequently decreases beyond 4.0 $\mu$m. On the basis of these results, we can establish the optimum operation point of the pump from the efficiency standpoint. To this end, we compute the ratio of the net flow rate to the work required. The flow rate is normalized by $a^3\omega$. The work is computed from the integral average of the product of the absolute values of the required forces normalized by $6\pi a^2\mu\omega$, and the particle velocity normalized by $a\omega$. We plot the results in figure 8. We see that the best efficiency is obtained with a microchannel of width 10 $\mu$m.

4. Two-lobe gear micro-pump

A two-lobe gear micro-pump designed by Terray et al [1] consists of two pairs of counter-rotating particles in a microchannel with a cavity, as shown in the sketch of figure 9. The geometry of the microchannel is 19 $\mu$m in $x_1$, 6 $\mu$m in $x_2$ and 4 $\mu$m in $x_3$. In the middle of the channel there is a cylindrical cavity with radius 3.5 $\mu$m. Two pairs of particles (two lobes) with radius $a = 1.5$ $\mu$m counter-rotate with a 90$^\circ$ phase angle offset.

As in the previous section, here too we use the inverse-problem approach to simulate the micro-pump; that is, we specify the motion and then compute and verify the forces to
generate the motion. To reproduce the counter-rotating motion for the particle pairs we studied the motion of the lobes in the videos provided in [1]. In particular, we first specify the motion for each pair of particles as a pure rotation around the center of mass of this particle pair, with a 90° phase angle offset for the two pairs of particles. The motion of the first particle of the top lobe is

\[ Y_1^1 = Y_{12} + a \cos[\pi - 2\pi t / T] \] (14)
\[ Y_2^1 = Y_{12} + a \sin[\pi - 2\pi t / T] \] (15)

while the motion of the second particle is

\[ Y_1^2 = Y_{12} + a \cos[-2\pi t / T] \] (16)
\[ Y_2^2 = Y_{12} + a \sin[-2\pi t / T] \] (17)

where \( Y_{12} \) is the coordinate of the center of mass of this lobe,

\( a \) is the particle radius and \( T \) is the period of the pump. The motion of the particles in the lower lobe is specified similarly.

From these analytic expressions we can compute the exact values of the target particle velocity \( V_T(t) \). To achieve this counter-rotational motion, a set of periodic forces \( F_{ext} \), in both the \( x_1 \) and \( x_2 \) directions for each particle, were computed via a penalty method from equation (11), as in the previous section, and included in the force monopole term of \( f(x, t) \) in equation (10). A similar iterative procedure as before was employed.

Figures 10 and 11 present the normalized computed forces on each particle in the \( x_2 \) and \( x_1 \) directions. Figure 12 shows the time evolution of the net flow rate \( Q(t) \) in the \( x_1 \) direction as a function of \( 2t / T \). After a short initial transient, the flow rate is established quickly and follows a time-periodic pattern. The direction of the net flow matches the observations from the experiments of Terray et al, and the non-dimensional averaged value \( Q / a^2 \omega \) is 1.483 which is in quantitative agreement with the results reported in [1].
Similar to the peristaltic micro-pump we study the effect of the width of the channel on the net flow rate. In figure 13 we plot the normalized forces and the corresponding flowrate as a function of the microchannel width. The results are similar to those of the peristaltic pump; however, the optimum operating point from the efficiency standpoint is at a width of $4 \mu m$. We also pursued a similar optimization study of a two-lobe gear pump inside a straight channel, see [12]. The results are similar to those of the two-lobe gear micro-pump with the same best operating point, i.e. at $4 \mu m$.

5. Summary and discussion

We have presented here the first three-dimensional simulations of the colloidal micro-pumps designed by Terray et al [1]. Both designs simulated here induce approximately the same net flow rate with the gear micro-pump giving slightly higher values. However, its drawback is the aggressive motion of its meshing gears that may affect adversely certain applications, e.g. transport of cells [1]. We have demonstrated a good agreement with the experimental results and have investigated ways of enhancing the performance of these micro-devices. We have also studied the stability of the flexible chain of cavity, see [12]. The results are similar to those of the two-lobe gear micro-pump with the same best operating point, i.e. at $4 \mu m$.
colloidal microspheres that provide the positive displacement that leads to net flow rate.

We have also investigated the robustness of the peristaltic pump in the presence of a parasitic force along the spanwise direction induced by the actuation, i.e. the optical trapping system. To this end, we introduce another traveling wave motion in $x_3$ with the same frequency as in $x_2$ but with an amplitude of 0.15 $\mu$m, that is 10% of the amplitude in $x_2$. Figure 14 compares the flow rate between disturbed and undisturbed flows in the $x_1$ and $x_3$ directions. We see that the 10% disturbance induces a slight deviation of about 0.3% in magnitude in the net flow rate. It also induces a small flow along the spanwise direction, which is non-zero instantaneously but its time-averaged value is zero.

From the simulation results, it is possible that a flow rate of 1 nl hr$^{-1}$ can be achieved at an excitation frequency of about 10 Hz. While optical trapping was used as the flow-activation element in [1], the use of paramagnetic beads excited remotely by a time-dependent magnetic field is another possible choice [13]. However, achieving the high flow rates in the higher frequency range that the simulation predicts may be challenging in the physical laboratory. Preliminary attempts to increase the frequency beyond 4 Hz in the experiments of Terray et al showed that the particles tended to fall out of the trap, thus disrupting the pumping motion [14]. Therefore, one may need to increase the laser power or use smaller size particles.

The force coupling method we have used has been validated systematically for prototype particulate microflows in [4, 5, 12]. Even at a frequency of 100 Hz the assumptions of linear Stokes flow are still applicable. The Reynolds number is about $10^{-3}$ and the scale $(2\mu/\rho \omega)^{0.5}$ for the Stokes layer associated with the oscillatory motion is still much larger than the channel width. We have also investigated the quality of simulation results based on the monopole term only, i.e. omitting intentionally the dipole term in equation (3). The predicted flow rate without the dipole is 7.5% less than that with the dipole. This suggests that the force dipole is important in simulating particulate flows with FCM even if the flow is in the Stokes regime. All the simulation results reported in this paper employed both the force monopole and dipole terms.

Acknowledgments

We would like to thank professor David W M Marr of Colorado School of Mines for helpful discussions. This work was supported by NSF and DARPA.

Appendix

In order to illustrate the flow structure in the channel and to verify the accuracy of the flow simulations with the FCM scheme, we compare the results with a full DNS simulation.

Figure 13. Gear micro-pump: optimization of the average flow rate with respect to the pump width in the $x_1$ direction.

Figure 14. Peristaltic micro-pump: variation of flow rate due to a parasitic force along the $x_3$ direction.
Figure 15. Flow velocity vectors in the $x_1, x_2$ plane at $t = 0$ from FCM computation: vectors indicate particle velocities, angular velocities are respectively $\Omega_3/\omega = 0.223, -0.009, -0.265, -0.002, 0.271, -0.047$.

Figure 16. Comparison of $u_1$ contours in the $x_1, x_2$ plane at $t = 0$ between DNS (top) and FCM (bottom) computations. Circles, radius $a$, show particle positions.

Figure 17. Comparison of $u_1$ contours in the $x_1, x_2$ plane at $t = 0$ between DNS (top) and FCM (bottom) computations. Circles, radius $1.2a$, show particle positions.

Figure 18. Comparison of $u_1$ velocity profiles at $t = 0$ between DNS (squares) and FCM (solid line): at $x_1 = 2$ (top) and at $x_1 = 7$ (bottom).

from the spectral element NEKTAR code [11]. For the computations with the FCM scheme, only a single spectral element decomposition of the flow domain is required. This consists typically of 128 elements, each with a polynomial expansion of between fourth and ninth order. The expansion order may be varied as needed to ensure accuracy. With a full simulation, a new spectral element decomposition of the flow domain is required in each time step to account for the instantaneous particle positions. The continuous generation of new meshes is very costly and not practical for routine simulations. Here, we compare flow results at a single instant, $t = 0$. The NEKTAR simulations were tested with 4088, 6950 and 10 162 elements with the expansion order from three to five.

Figure 15 shows the flow velocity vectors at $t = 0$, as computed from the FCM scheme. The height of the channel has been expanded to 6.4 $\mu$m to accommodate the mesh of the full DNS. The flow is shown for the $x_1, x_2$ plane passing through the centers of the particles. The velocity of each particle is indicated. The particles are also rotating and the components of the angular velocity $\Omega_3$ are non-zero. The flow vectors show how the motion of the particles forces a resultant net flow, from right to left, in the negative $x_1$ direction.

The prescribed particle velocities, together with the angular velocities $\Omega^{(n)}$ computed from (8), are used to set
the boundary conditions for the full NEKTAR simulations. Figure 16 shows the contours of the velocity component $u_1$ computed at $t = 0$ from both FCM and NEKTAR. There is generally a good agreement between the two, except for regions close to the particle surface. A further comparison is shown in figure 17, where the circles denoting the particles have been expanded by 20% to eliminate the near-surface region. A more detailed comparison is shown in figure 18 with the velocity profiles at two different locations in the mid-plane of the channel, $x_1 = 2$ that lies to the left of the particles, and $x_1 = 7$ that coincides with the initial position of the first particle. Again there is similar agreement between the two simulations, with better agreement away from the particle surface.

The results for the flow data and the forces, in general, may be specified in a non-dimensional form by using the scales based on the particle radius $a$ and the angular frequency of the oscillations $\omega$. As the underlying equations for Stokes flow are linear, the flow velocities scale with $a\omega$ and the net volume flow rate $Q(t)$ through any section of the microchannel scales with $a^3\omega$. For the flows computed above, at $t = 0$, the values of $Q/a^3\omega$ are between $-1.57$ and $-1.63$ for the DNS results, depending on the resolution and the number of elements used. For the FCM computations the corresponding flow rate is $-1.57$.

References

[14] Marr D 2003 Personal communication