SIMULATIONS OF FLOW OVER A FLEXIBLE CABLE: A COMPARISON OF FORCED AND FLOW-INDUCED VIBRATION

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We present direct numerical simulation results from an on-going investigation of flow over a flexible cable. In particular, we compare forced cable vibration with flow-induced cable vibration at Reynolds numbers 100, 200 and 300. The cables are assumed to be infinitely long and periodic—we choose two spanwise wavelength cases: the “short” case ($L/d = 12.6$), and the “long” case ($L/d = 45$). We first examine the dynamics of the cable, concentrating on lift forces and the power produced by these lift forces. We then examine the flow structures in the wake of the vibrating cable. We observe a breakdown of a time-periodic cable and wake response going from $Re = 100$ to $Re = 200$. At $Re = 100$ we observe relatively similar responses comparing the forced vibration case with the flow-induced vibration case, but at $Re = 200$ differences become larger. Finally, we present a comparison with the experimental results of forced vibration by Ramberg and Griffin.

1. INTRODUCTION

Fluid flows over flexible cables arise in many engineering situations, such as marine cables towing instruments, flexible risers used in petroleum production and mooring lines—see Blevins (1977), Vandiver (1991) and Ramberg & Griffin (1976) for overviews of this problem. It is therefore important to understand and be able to predict the hydrodynamic forces and motion of cables caused by flow-induced vibration. In Figure 1 we plot results from several experiments on flow–structure interactions [results compiled by Griffin (1995)]. The plot shows peak-to-peak vibration amplitude (nondimensionalized by diameter) versus “reduced damping” (which is proportional to the product of the cable mass and damping). We see that, for low values of reduced damping, the vibration amplitudes are around two diameters peak-to-peak. We also show results from two-dimensional simulations we performed for two different mass ratios. The simulations underpredict the vibration amplitude at low values of reduced damping, and overpredict the vibration amplitude at large values of reduced damping. This discrepancy leads us to the question of why the two-dimensional simulations do not accurately predict the vibration amplitude, and whether three-dimensional simulations will more closely match the experimental results.

In this paper we show results of direct numerical simulations of flow over a flexible cable. Cable motion, as well as lift and drag forces on the cable are computed. We are interested in comparing the differences between forced cable vibration with flow-induced cable vibration. Two sets of simulations were performed. In the first set we considered flow-induced vibration and forced vibration with a short wavelength
Figure 1. Peak-to-peak flow-induced vibration amplitude versus reduced damping (mass-damping parameter); data compiled by Griffin (1995). Also shown are the results of simulations: · · · · , $m/\rho d^2 = 1$; · · · · · · , $m/\rho d^2 = 10$.

($L/d = 12.6$) at Reynolds number 100 and 200. In the second set we considered forced vibration with a long wavelength ($L/d = 45$) at Reynolds numbers 100, 200 and 300. In the first set (short wavelength vibration) we chose forced vibration frequency and amplitude to match that measured in the flow-induced vibration case. In the second set (long wavelength vibration) we set the forced vibration frequency and amplitude to match the frequency and amplitude used in the experiment of Ramberg & Griffin (1976).

2. FORMULATION

2.1. GOVERNING EQUATIONS

We consider the interaction of an incompressible fluid flowing past a long flexible cable under tension. The equations that describe this problem are the coupled system of fluid equations and cable equations. The fluid equations are given by the Navier–Stokes equations and the continuity equation. In a stationary, Cartesian coordinate system ($x', y', z'$) these equations are

$$\frac{\partial u'}{\partial t} + (u' \cdot \nabla)u' = -\nabla p + \nu \nabla^2 u', \nabla \cdot u' = 0, \quad (1)$$

For forced cable vibration, the motion of the cable is prescribed, usually in the form of a standing wave (for simplicity, constrained to move only in the cross-stream direction) with an amplitude $A$, wavelength $L$, and frequency $\omega_f$:

$$\zeta(z, t) = A \cos(\omega_f t) \cos(2\pi z/L). \quad (2)$$

In the case of flow-induced vibration, the equation of motion of the cable for its two directions of motion (i.e. in the $x$- and $y$-directions) is given by a slightly modified forced wave equation:

$$\frac{\partial^2 \xi}{\partial t^2} - \frac{\partial^2 \xi}{\partial z^2} - \omega^2 \xi + \frac{1}{m} F(z, t), \quad (3)$$
where $\xi(z, t) = (\zeta(z, t), \eta(z, t))$ gives the cable displacement in the streamwise and cross-stream directions and $c = \sqrt{T/m}$ gives the phase speed of waves in the cable. The cable has mass per unit length $m$ and tension $T$. To maintain a mean displacement, the cable is lightly elastically supported by linear springs with spring constant $k$, giving a natural frequency of $\omega = \sqrt{k/m}$. The spring constant is selected sufficiently small to have negligible effect on the cable response. In this case the value of $k$ was chosen such that the natural frequency of the first mode of cable vibration was affected by less than 5%. The fluid force on the cable is denoted by $F(z, t)$. The components of $F(z, t)$ in the streamwise and cross-stream directions are the drag and lift force on the cable. Internal damping is neglected here as its does not significantly influence the response, as is evident from the experimental results (Figure 1).

To simplify the solution of the fluid equations, we use a coordinate system attached to the cable. This maps the time-dependent and deforming problem domain to a stationary and non-deforming one as shown in Figure 2. This mapping is described by the following transformation:

$$
x = x' - \zeta(z', t'),
$$
$$
y = y' - \eta(z', t'),
$$
$$
z = z'.
$$

Accordingly, the velocity components and pressure are transformed as follows:

$$
u = \nu' - \frac{\partial z'}{\partial t}, \quad \frac{\partial \zeta}{\partial z'},
$$
$$
\nu = \nu' - \frac{\partial \eta}{\partial t} - \omega' \frac{\partial \eta}{\partial z'},
$$
$$
w = w', \quad \rho = p'.
$$

In the transformed system of coordinates the cable appears as a straight and stationary cable (see Figure 2). The Navier–Stokes equation and continuity equation (1) are transformed as follows:

$$
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{A}(\mathbf{u}, p, \xi),
$$
$$
\nabla \cdot \mathbf{u} = 0.
$$
The forcing term \( \mathbf{A}(u, p, \xi) \) is the extra acceleration introduced by transformation (5). The \( x, y \) and \( z \) components of \( \mathbf{A} \) have inviscid and viscous parts and are given by:

\[
A_x = -\frac{d^2 \zeta}{dt^2} + v \left[ \frac{\partial}{\partial z^2} \left( u + \frac{\partial \xi}{\partial z} w \right) - \frac{\partial}{\partial z} u \right] + v \left[ \frac{\partial \xi}{\partial z} \nabla_{xy} w + \frac{\partial \xi}{\partial t} \frac{\partial \xi}{\partial z} \right],
\]

\[
A_y = -\frac{d^2 \eta}{dt^2} + v \left[ \frac{\partial}{\partial z^2} \left( v + \frac{\partial \eta}{\partial z} w \right) - \frac{\partial}{\partial z} v \right] + v \left[ \frac{\partial \eta}{\partial z} \nabla_{xy} w + \frac{\partial \eta}{\partial t} \frac{\partial \eta}{\partial z} \right],
\]

\[
A_z = \frac{\partial \xi}{\partial z} \frac{\partial p}{\partial x} + \frac{\partial \eta}{\partial z} \frac{\partial p}{\partial y} + v \left[ \frac{\partial}{\partial z^2} w - \frac{\partial}{\partial z} w \right],
\]

where

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z},
\]

\[
\frac{\partial}{\partial z^2} = \frac{\partial}{\partial z} \frac{\partial}{\partial x} - \frac{\partial}{\partial z} \frac{\partial}{\partial y},
\]

\[
\nabla_{xy} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.
\]

2.2. NUMERICAL METHOD

To solve the three-dimensional Navier–Stokes equations, we used a parallel spectral element/Fourier method, developed by Henderson & Karniadakis (1995). Spectral elements are used to discretize the \( x-y \) planes, while a Fourier expansion is used in the \( z \)-direction (i.e. along the cable). Consequently all variables are assumed to be periodic in the spanwise direction. Each Fourier mode is assigned to a separate processor, allowing efficient parallel computation. Computing one time-step of the combined cable/flow problem proceeds as follows: first the nonlinear terms including the extra terms due to the cable motion are evaluated explicitly in time; next the pressure is computed from a Poisson equation; then the viscous terms are computed implicitly in time via a Helmholtz solver using the most recent velocity boundary conditions; finally, the fluid forces on the cable are computed, and the cable motion and velocity boundary conditions are updated.

The computational domain extends \( 35d \) (cable diameters) downstream, and \( 15d \) above and below the cable. The cable is assumed to be infinitely long, and periodic. The “cable span”, i.e. wavelength of the cable was \( L/d = 12.6 \) for the first set of simulations (short wavelength vibration), and \( L/d = 45 \) for the second set of simulations (long wavelength vibration). Each \( x-y \) plane is discretized by a 110 element mesh, with each element having 81 collocation points. Typically, 32 \( z \)-planes (16 Fourier modes) are used giving a total of 225,000 mesh points. A nondimensional time step of \( U \Delta t/d = 0.002 \) is used giving over 3000 time steps per shedding cycle. Computation time on a 16-node IBM-SP2 is about 1 hour per shedding cycle. For each simulation, the cable initial position and velocity are set and the simulations are run for at least 20 shedding cycles, or until the statistics are relatively stationary. For all the forced, and the first standing-wave simulation, cable motion was restricted to the cross-stream direction, since this is the primary direction of cable motion. In the flow-induced vibration simulations at \( Re = 200 \), the cable was free to move in both the streamwise and cross-stream directions.
3. SHORT WAVELENGTH VIBRATION

3.1. REYNOLDS NUMBER OF 100

In this section we consider two cases: one flow-induced vibration case, and one forced vibration case. The flow-induced vibration case comes from an initial cable position of a standing wave. In this case the cable is constrained to only allow motion in the cross-stream direction. The flow-induced vibration response is periodic for many shedding periods. Eventually, in the absence of any end conditions to the cable (as in the case with the periodic assumption), this cable response would become a traveling wave. The forced vibration case is chosen to match the flow-induced vibration case of a standing-wave response, with no cable motion in the streamwise direction. Note that we will use the phrases “free vibration” and “flow-induced vibration” interchangeably.

For all the cases, we need to choose the appropriate cable parameters: cable phase speed \( c \) for the free vibration, and vibration amplitude and frequency for the forced vibration. To determine an appropriate cable phase speed for free vibration, we consider simple wave solutions to the wave equation. The simplest periodic solutions to the wave equation are standing waves and traveling waves. Both a standing-wave response, \( y(z, t) = A \cos(\omega t) \cos(2\pi z/L) \), and a traveling wave response, \( y(z, t) = A \cos(\omega t - 2\pi z/L) \), satisfy the wave equation, \( y_{tt} = c^2 y_{zz} \), when \( \omega = 2\pi c/L \). We considered one period of a cable spanwise length of \( L/d = 4\pi \approx 12.6 \). Assuming that the forcing due to vortex shedding is at a frequency \( \omega \), the appropriate cable tension is computed using \( c = \omega L/(2\pi) \) where \( c = \sqrt{T/m} \).

In the case of \( \text{Re} = 100 \), two-dimensional results show that the Strouhal number \( \text{St} = f d/U \) is approximately 0.167 which corresponds to a vortex-shedding frequency \( \omega = 1.05 \), so the phase speed is computed as \( c = 2.1 \). An initial cable amplitude of 0.5 diameter was specified. For the free vibration cases, the simulation was started with the cable having the appropriate initial displacement and velocity in the cross-stream direction. The simulations were run for more than 20 shedding cycles, until a time-periodic state was reached, and a constant amplitude response was obtained.

For the forced cable case, the amplitude and frequency parameters were chosen to
match those for the free vibration standing-wave case. Therefore, the amplitude was chosen as $A = 0.7$ diameter, and the frequency as $\omega_f = 1.00$ to match the frequency observed in the free vibration traveling wave cable response.

Let us first examine the cable responses in the free vibration cases. We plot cross-stream displacement $y/d$, lift coefficient $C_l$ (given a lift force $F_l$, we defined $C_l = 2F_l/(pdLU^2)$), nondimensional power produced by lift force $W$, defined as $W = C_l y U$, and finally drag coefficient, $C_d$. Note that the (nondimensional) power produced by the lift force, $W$, is a measure of the instantaneous energy transfer between the cable and fluid. These variables are plotted versus time $tU/d$ (horizontal axis) and distance along cable $z/d$ (vertical axis). Figure 4 shows these four contour plots for the standing-wave (constrained) free vibration response, and the forced vibration response. Dotted contour levels denote a negative value. In both figures, we see approximately three shedding periods (~20 time units) displayed, showing the time-periodic response. The vibration frequency is approximately the same in both the free and forced cases. The top plots in each case show the cross-stream cable vibration amplitude of approximately 0.7 cable diameter. Here, it is obvious that we are

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{Cable cross-stream displacement $y/d$, lift coefficient $C_l$, nondimensional power $W$ and drag coefficient $C_d$ versus time $tU/d$ and distance along cable $z/d$ for flow-induced vibration (left) and forced vibration (right), both at Re = 100.}
\end{figure}
observing a standing-wave cable response. Moving on to the lift coefficient plots, we see that the largest lift coefficient in the free vibration case is similar to that in the forced vibration case. However, the power produced in both cases is approximately the same, due to the location of maximum lift not occurring at the antinodes of the standing wave. Finally, we see that the maximum drag coefficient is approximately the same, but the minimum is lower in the free vibration case. Note that the mean drag coefficient is about 40% larger in both cases compared to the two-dimensional drag coefficient of approximately $C_d = 1.4$ for flow over a fixed cylinder at $Re = 100$. The forced standing-wave vibration case shows a slightly lower power and slightly higher drag coefficient compared to the free standing-wave vibration case.

These results are summarized in Table 1. We see that the maximum values for the displacement, lift and drag coefficient are all observed in the standing-wave case. We also include for reference results from corresponding 2-D simulation of free vibrations.

We now look at the flow structures in the wake of the cable. The simplest way to visualize the flow in the wake is to look at iso-surfaces of spanwise vorticity. In the two-dimensional case for a fixed cylinder, this will show the familiar von Karman vortex street. Figures 5 and 6 show iso-surfaces of equal and opposite signs of spanwise vorticity (viewed from above) for the standing-wave free vibration case and standing-wave forced vibration case. The cable is located at $x = 0$, and the flow is from left to right. The antinodes of the standing wave are located at the two ends and middle of the cable in the figure. Note that for a fixed cylinder with periodic boundary conditions in the span, at this Reynolds number ($Re = 100$), we would observe parallel shedding, i.e., from this view, parallel bands of the two signs of vorticity. With the cable vibrating, however, we see very different wake patterns depending on cable response: a standing wave produces a staggered pattern of vorticity that is connected in a woven-lattice pattern. Note that the standing-wave cable motion produces a very similar pattern, irrespective of whether the standing wave was forced, or was a result of flow-induced vibration. The other possible flow-induced cable response at this Reynolds number ($Re = 100$) is a traveling wave, which produces oblique shedding—these results are reported by Newman & Karniadakis (1996). The vorticity strength remains larger downstream in the oblique shedding case. This is because the vortical structure is more coherent (i.e., not interwoven), as it is in the wake of the standing wave. A travelling wave cable response produces oblique shedding of spanwise vorticity, i.e., much like the shedding seen in the fixed cylinder case, but at an angle to the spanwise direction. Oblique shedding has been observed in flows over fixed cylinders under certain experimental conditions by Williamson (1989) and Hammache & Gharib (1991).

We can get more insight into the structure of the flow in the wake of these vibrating

<table>
<thead>
<tr>
<th>Case</th>
<th>$y/d$</th>
<th>$C_l$</th>
<th>$min C_d$</th>
<th>$max C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>—</td>
<td>0.34</td>
<td>1.36</td>
<td>1.38</td>
</tr>
<tr>
<td>2-D free</td>
<td>0.52</td>
<td>0.20</td>
<td>1.62</td>
<td>2.08</td>
</tr>
<tr>
<td>Standing</td>
<td>0.69</td>
<td>0.50</td>
<td>1.34</td>
<td>2.26</td>
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</table>
cables by looking at slices of the flow perpendicular to the cable (i.e., in the $x$–$y$ plane). Again, at $Re = 100$, flow over a cylinder is two-dimensional and we see the well-known von Karman vortex street pattern of alternating signs of spanwise vorticity. Starting with this familiarity of the wake structure in this simple case, we now look at iso-contours of spanwise vorticity in the wake of the vibrating cables. Figure 7 shows spanwise vorticity at slices taken at four evenly spaced intervals along one wavelength of the cable for the two $Re = 100$ cases. The first and third plots show slices behind the cable antinodes, while the second and fourth plots show slices behind the cable nodes, as marked. The same contour levels are used in all plots for easy comparison, and dotted contour levels are negative.

We see quite a different structure to the spanwise vorticity going along the span of the cable. There is a large difference in the slices at nodes and antinodes. The streamwise and cross-stream spacing of vortices at antinodes is larger than the spacing at nodes. This typically leads to a vortex pairing further downstream as has been observed experimentally by Nakano & Rockwell (1993). Here, however, due to the relatively short domain and low Reynolds number, such a pairing was not observed.

In this more detailed view of spanwise vorticity, we see slight differences in the wake produced by the flow-induced standing wave versus the forced standing wave in Figure 7. First of all, we see a degradation of the anti-symmetric shedding pattern observed behind the nodes of the forced standing wave. Furthermore, we see a somewhat more irregular pattern of vorticity behind the antinodes of the forced standing wave.
3.2. Reynolds Number of 200

In this section we consider two cases: one flow-induced vibration case, and one forced-vibration case. The cable in the flow-induced vibration case is allowed to move in both the streamwise and cross-stream directions. The constrained standing-wave case was not run for Re = 200, because at this Reynolds number the standing-wave response becomes primarily a traveling wave response. The cable parameters are the same as the ones used for Re = 100, i.e., phase speed of $c = 2 \cdot 1$ and forced-vibration amplitude and frequency of $A = 0.7$ diameters and $\omega_f = 1.00$, respectively. We decided to fix the forced-vibration amplitude and frequency at the same values to enable direct comparison with the Re = 100 case.

We start by examining the cable response in the free-vibration case and forced-vibration case at Re = 200. Figure 8 shows the cross-stream displacement, lift coefficient, power produced by lift force and drag coefficient versus time and distance along the cable. We immediately see the breakdown of time-periodic cable dynamics (except for the forced standing-wave cross-stream displacement, which is prescribed). In the case of the flow-induced vibration, we are observing primarily a traveling wave.

![Figure 8. Cable cross-stream displacement $y/d$, lift coefficient $C_l$, non-dimensional power $W$ and drag coefficient $C_d$ versus time $tU/d$ and distance along cable $z/d$ flow-induced vibration (left) and forced vibration (right), both at Re = 200.](image)
Figure 11. Slices of spanwise vorticity downstream from four equal spaced points on the flow-induced vibrating cable (left) and forced vibrating cable antinodes and nodes (right), both at Re = 200.

Note that the range of amplitudes is approximately two cable diameters peak-to-peak (consistent with the experimental results in Figure 1), larger than the roughly constant 0.7 diameters observed in the Re = 100 free-vibration case. Somewhat more dramatic are the significantly larger forces, indicated by the lift coefficient and drag coefficient plots.

Let us now compare these plots to the corresponding plots for the Re = 200 forced-vibration case. First, note that despite the regular motion of the cable, the force coefficients are less regular—particularly the lift coefficient. The drag coefficient is more “locked into” the motion of the cable. We observe slightly larger lift forces in the free case, and significantly larger drag forces.

Let us now consider the iso-surfaces of spanwise vorticity in the wake of the free-vibration and forced vibration Re = 200 cases in Figures 9 and 10. Here we see the breakdown of the time-periodic laminar wake. In the free vibration case we observe indications of oblique shedding due to the traveling wave response of the cable. In this case the flow is more three-dimensional than the corresponding flow in the Re = 100 case. In the forced vibration case, we note that the pattern in the wake does not show the strong staggered pattern as in the Re = 100 case. In fact we observe a wake structure somewhat similar to the one observed in the free vibration case, although the level of spanwise vorticity is lower in the forced vibration case.

The slices of spanwise vorticity, plotted in Figure 11 for free (left plots) and forced (right plots) vibration, show that the wake is far more three-dimensional at Re = 200. The four slices along the cable in the free vibration case all show similar features. However, in the case of the forced standing wave, we do see a wider wake behind the
antinodes compared to the width of the wake behind the nodes, although this is not as
dramatic as the difference in the wake width for the standing-wave case at \( \text{Re} = 100 \). This may be due to the different amplitudes. These results are summarized in Table 2.

### Table 2

Results for flow-induced (free) vibration versus forced vibration at
\( \text{Re} = 100 \) and \( \text{Re} = 200 \) for \( L/d = 12.6 \)

<table>
<thead>
<tr>
<th>Type</th>
<th>Re</th>
<th>( y/d )</th>
<th>( \dot{C}_d )</th>
<th>( C'_d )</th>
<th>( C_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>100</td>
<td>0.7</td>
<td>1.8018</td>
<td>0.4648</td>
<td>0.4963</td>
</tr>
<tr>
<td>Forced</td>
<td>100</td>
<td>0.7</td>
<td>1.8609</td>
<td>0.5027</td>
<td>0.6210</td>
</tr>
<tr>
<td>Free</td>
<td>200</td>
<td>1.05</td>
<td>2.0339</td>
<td>1.5681</td>
<td>1.5506</td>
</tr>
<tr>
<td>Forced</td>
<td>200</td>
<td>0.7</td>
<td>1.7955</td>
<td>0.6865</td>
<td>1.3416</td>
</tr>
</tbody>
</table>

4. LONG WAVELENGTH VIBRATION

Experiments examining the effects of vortex coherence on the flow-induced forces on
vibrating cables were performed by Ramberg and Griffin at NRL, and their results
were reported in Ramberg & Griffin (1976). We simulated one of the cases described in
that experiment. The case was a forced standing wave, with wavelength \( L/d = 45 \)
diameters, amplitude \( y/d = 0.15 \) diameters, and frequency 92\% of the shedding
frequency for the fixed cylinder, i.e., \( \omega_f = 1.27 \). The experiment was at \( \text{Re} = 570 \), while
we conducted simulations at \( \text{Re} = 100 \) and \( \text{Re} = 200 \) using an amplitude of \( y/d = 0.3 \)
diameters, and at \( \text{Re} = 300 \) using an amplitude of \( y/d = 0.15 \) diameters. Figures 12, 13
and 14 show the cable cross-stream displacement \( y/d \), lift coefficient \( C_l \),
nondimensional power \( W \) and drag coefficient \( C_d \) versus time \( tU/d \) and distance along
cable \( z/d \) for these forced vibration cases at \( \text{Re} = 100, 200 \) and 300, respectively. We
see relatively similar results in the Reynolds number 100 and 200 cases; differences
include slightly higher lift coefficients at \( \text{Re} = 100 \) and slightly lower drag coefficients at
\( \text{Re} = 200 \). Observe that the forces on the cable are time-periodic at \( \text{Re} = 100 \), but not
so at \( \text{Re} = 200 \), particularly at the location of the nodes of the standing wave. Note that
at both \( \text{Re} = 100 \) and 200, the instantaneous power developed by the lift forces is
positive, i.e., energy is being transferred from the fluid to the cable. Now consider the
\( \text{Re} = 300 \) case. We see in this case (recalling that the cable vibrations are half the
amplitude as was for \( \text{Re} = 100 \) and 200) the lift forces are significantly higher, which is
due to the smaller motion of the cable. The drag coefficient for \( \text{Re} = 300 \) is
approximately the same as that for \( \text{Re} = 200 \), however the power due to lift forces has a
much smaller magnitude, partly due to the smaller motion of the cable.

The vortical structures in the wake of this long vibrating cable (\( L/d = 45 \)) look quite
different to those observed in the short wavelength vibrating cable (\( L/d = 12.6 \)). One
major factor that affects the structures observed is the curvature or angles of the cable
“seen” by the incoming flow. In the short wavelength case, say for the simulated
amplitude of \( y/d = 0.7 \), the height to wavelength ratio is \( y_{\text{max}}/L = 0.056 \) while in the
long wavelength case this height to wavelength ratio is \( y_{\text{max}}/L = 0.0067 \), one order of
magnitude smaller. Figure 15 shows a top view of equal and opposite signs of spanwise
vorticity for Reynolds numbers of 100, 200 and 300. The darker shade is negative
vorticity, and the lighter shade is positive vorticity. There are several features to note.
First note that we do not see a staggered pattern of vorticity as is seen in Figure 5.
Secondly, we do not see the symmetric shedding pattern (symmetry about the nodes or antinodes) observed in the short wavelength case. We also see a bowing of the rolls of vorticity as they convect downstream (Re = 100), and a more three-dimensional, turbulent wake behind the nodes (Re = 200 and 300). Also notice how the disturbances initiated behind the nodes propagate downstream with a spanwise component.
Figure 13. Cable cross-stream displacement y/d, lift coefficient $C_l$, nondimensional power $W$ and drag coefficient $C_d$ versus time $tU/d$ and distance along cable $z/d$ for $L/d = 45$; forced vibration, Re = 200.

We finally examine the spanwise vorticity field even closer by taking slices perpendicular to the cable at the location of the cable standing-wave nodes and antinodes. Figure 16 shows these slices for the long wavelength cable forced vibration at Re = 100 and 200, while Figure 17 show slices for the Re = 300 case and a comparison of flow visualizations taken from the Ramberg & Griffin experiment at the
Figure 14. Cable cross-stream displacement $y/d$, lift coefficient $C_l$, nondimensional power $W$ and drag coefficient $C_d$ versus time $tU/d$ and distance along cable $z/d$ for $L/d = 45$; forced vibration, Re = 300.

same spanwise locations. We see that there is again a dramatic difference in the vortex street in this long wavelength case compared with the corresponding plots in the short wavelength case (Figure 7). Starting with Re = 100, we do not see much difference in the vortex streets at different positions along the cable. However, at Re = 200, we see a definite difference in the flow patterns behind the node compared with the patterns
behind the antinode, somewhat more defined that the corresponding case for the short wavelength case (Figure 11). We observe a qualitative agreement with the experimental results, with the wake being out of the lock-in state at the node \((a/d = 0)\), in transition at \(a/d = 0.09\) (0.075 of one wavelength), and in a lock-in state at the antinode \((a/d = 0.3)\). In the experiment, they denote peak-to-peak amplitude as \(a\) (i.e., \(a = 2y\)).

5. SUMMARY

We have performed a series of direct numerical simulations for flexible cables to quantify the coupled cable-flow response in the low Reynolds number regime. Both short and long wavelength cable vibrations were considered, and the effects of the transition from laminar to transitional flows were investigated. The comparison of forced versus flow-induced vibrations of cables reveals that the responses may not be similar, and that these differences increase at higher Reynolds numbers. A tentative conclusion from this ongoing investigation is that transition in the wake occurs at lower Reynolds numbers in the case of the flow-induced vibration, whereas forced vibration seems to somewhat delay this transition.

ACKNOWLEDGEMENTS

This paper is dedicated to the memory of our colleague O. M. Griffin with whom we had very fruitful collaborations over the last few years.

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Figure 5. Top view of equal and opposite signs of spanwise vorticity. Flow-induced vibration, $Re = 100$.

Figure 6. Top view of equal and opposite signs of spanwise vorticity. Forced vibration, $Re = 100$. 

Figure 9. Top view of equal and opposite signs of spanwise vorticity. Flow-induced vibration, Re = 200.

Figure 10. Top view of equal and opposite signs of spanwise vorticity. Forced vibration, Re = 200.
Figure 15. Top view of equal and opposite signs of spanwise vorticity: $L/d = 45$ forced vibration for (a) $Re = 100$, (b) $Re = 200$ and (c) $Re = 300$. 
Figure 16. Slices of spanwise vorticity downstream from vibrating cable anti-nodes and nodes for $L/d = 45$; forced vibration at (a) $Re = 100$ and (b) $Re = 200$.

Figure 17. Comparison of slices of spanwise vorticity for $L/d = 45$ forced vibration showing $Re = 570$ experiment (left) and $Re = 300$ simulation (right). Point $a/d = 0$ is forced vibration standing-wave node, and point $a/d = 0.3$ is forced vibration standing-wave antinode.