Homework 3

Consider the one-dimensional Helmholtz equation in cylindrical coordinates:

\[
\frac{1}{r} \frac{d}{dr} \left[ r \frac{du}{dr} \right] - u = 0
\]

where \( u(r = 0) \) is bounded and \( u(r = 1) = 1 \).

1. Find the functional for the above problem and the corresponding variations at the end points.

2. Compute a numerical solution to this problem using linear finite elements. Plot the error in norms \( L_2 \) and \( H^1 \) versus the number of elements and compare with the known error estimates. (Note: Find the exact solution first).