Effects of Shock Waves on Rayleigh-Taylor Instability

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**ABSTRACT**

A numerical simulation of two dimensional compressible Navier-Stokes equations using a high order weighted essentially non-oscillatory (WENO) finite difference shock capturing scheme is carried out in this paper, to study the effect of shock waves on the development of Rayleigh-Taylor instability. Shocks with different Mach numbers are introduced ahead or behind the Rayleigh-Taylor interface, and their effect on the transition to instability is demonstrated and compared. It is observed that shock waves can speed up the transition to instability for the Rayleigh-Taylor interface significantly. Stronger shocks are more effective in this speed-up process.

**Key Words:** Navier-Stokes equations; Rayleigh-Taylor instability; WENO scheme, high order accuracy, numerical simulation

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1 Introduction

In this paper, we are interested to study the effect of shock waves on the development of Rayleigh-Taylor instability, through a numerical simulation of the two dimensional Navier-Stokes equations using a high order weighted essentially non-oscillatory (WENO) finite difference shock capturing scheme.

Rayleigh-Taylor instability (RTI) results from the application of a pressure gradient (e.g. a gradient due to gravity) in the direction opposite to a density gradient \([1, 2, 3, 4]\). It happens on an interface between fluids with different densities when an acceleration is directed from the heavy fluid to the light fluid. The instability has a fingering nature, with bubbles of light fluid rising into the ambient heavy fluid and spikes of heavy fluid falling into the light fluid, see for example \([5, 6]\). Eventually, a flow induced by RTI will develop into a high Reynolds number turbulence with very strong nonlinearity \([7]\). The turbulent flows induced by RTI have found a wide range of applications both in an astrophysical setting \([8, 9]\) and in an inertial confinement fusion \([10]\). Progress in an understanding of RTI induced flows \([11]\) will hopefully lead to improved models for astrophysical and engineering calculations \([12, 13]\).

When an external agency is present, the evolution of a RTI induced flow will be significantly modified. Carnevale et al. \([14]\) demonstrated that the growth of the mixing zone generated by RTI can be greatly retarded by the application of rotation. Specifically, for a weak rotation, the development of the mixing zone would not proceed as far as that of a non-rotating case. For a strong rotation, however, the growth of the perturbation would be diminished so significantly that there is little that can be identified as a mixing layer formation. Pacitto et al. \([15]\) reported a RTI experiment using a magnetic fluid and applying a magnetic field. These authors measured different values of the magnetic field, the wavelength, and the growth rate of the observed pattern. The magnetic field was found to destabilize the interface, decrease the wavelength, and increase the growth rate.

The objective of this paper is to study the effect of shock waves on the development of
the Rayleigh-Taylor instability. Shocks with different Mach numbers are introduced ahead or behind the Rayleigh-Taylor interface, and their effect on the transition to instability is demonstrated and compared. It is observed that shock waves can speed up the transition to instability for the Rayleigh-Taylor interface significantly. Stronger shocks are more effective in this speed-up process.

2 Numerical method and RTI setup

Numerical experiments are performed using a ninth order finite difference WENO scheme [16] associated with an eighth-order central approximation to the viscous terms, for the two dimensional non-dimensionalized Navier-Stokes equations with gravitation source terms:

\[
\begin{align*}
\rho_t + (\rho u)_x + (\rho v)_y &= 0, \\
(\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y &= \frac{1}{Re} \left( \frac{4}{3} u_{xx} + u_{yy} + \frac{1}{3} v_{xy} \right), \\
(\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y &= \frac{1}{Re} \left( v_{xx} + \frac{4}{3} v_{yy} + \frac{1}{3} u_{xy} \right) + \rho, \\
E_t + [u(E + p)]_x + [v(E + p)]_y &= \frac{1}{Re} \left( \frac{2}{3} (u^2)_{xx} - \frac{2}{3} (uv)_x + \frac{1}{2} (v^2)_{xx} + (vu)_x ight) \\
&+ \frac{1}{2} (u^2)_{yy} + (uv)_y + \frac{2}{3} (v^2)_{yy} - \frac{2}{3} (wu)_y \\
&+ \frac{1}{(\gamma - 1) Pr} \left( (C^2)_{xx} + (C^2)_{yy} \right) + \rho u,
\end{align*}
\]

where \( \rho \) is the density, \((u, v)\) is the velocity, \( E \) is the total energy, and \( p \) is the pressure, related to the total energy by \( E = \frac{\rho}{\gamma - 1} + \frac{5}{2} \rho (u^2 + v^2) \) with the ratio of specific heats \( \gamma \) being a constant. \( C \) is the sound speed satisfying \( C^2 = \frac{\gamma p}{\rho} \). \( Re \) is the Reynolds number. \( Pr = 0.7 \) is the Prandtl number.

The class of high order finite difference WENO schemes, coupled with total variation diminishing (TVD) high order Runge-Kutta time discretizations [17], was developed in [18] for the fifth order accurate version and in [16] for the higher order versions, including the
ninth order version that we use in this paper. The resolution of these high order WENO schemes when applied to high Reynolds number Navier-Stokes equations has been studied in detail in [19]. It was shown in [19] that it is advantageous in terms of CPU time to use a higher order WENO scheme to simulate flows with both shocks and complicated smooth flow features, such as the problem of Rayleigh-Taylor instability under study. This is the motivation for us to choose the ninth order WENO scheme [16] for the simulation in this paper. The Reynolds number in the Navier-Stokes equations (2.1) is taken as $Re = 25,000$, which requires a mesh size $h = \frac{1}{240}$ in order to obtain resolved numerical solutions according to the study in [19].

We setup the RTI as follows. The computational domain is $[0, \frac{1}{2}] \times [0, 1]$. Initially the interface is at $y = \frac{1}{2}$. The heavy fluid with density $\rho = 2$ is below the interface, and the light fluid with density $\rho = 1$ is above the interface with the acceleration in the positive $y$ direction. The pressure $p$ is continuous across the interface. A small perturbation is given to the $y$-direction fluid speed; thus for $0 \leq y < \frac{1}{2}$, $\rho = 2$, $u = 0$, $p = 2y + 1$, $v = -0.025C \cdot \cos(8\pi x)$, and for $\frac{1}{2} \leq y \leq 1$, $\rho = 1$, $u = 0$, $p = y + \frac{3}{2}$, $v = -0.025C \cdot \cos(8\pi x)$, where $C$ is the sound speed, $C = \sqrt{\frac{\gamma}{\rho}}$, and the ratio of specific heats $\gamma = \frac{5}{3}$. Reflective boundary conditions are imposed for the left and right boundaries. At the top boundary, the flow values are set as $\rho = 1$, $p = 2.5$, $u = v = 0$, and at the bottom boundary, they are set as $\rho = 2$, $p = 1$, $u = v = 0$. The source term $\rho$ is added to the right hand side of the third equation and $\rho v$ is added to the fourth equation in the Navier-Stokes system (2.1).

In Figure 2.1, the Rayleigh-Taylor flow at $T = 1.85$, 2.1 and 2.5 are shown. This should serve as a reference to compare with the results in next section when shock waves are introduced into the flow.

3 Simulated RTI flow fields and Shock waves

We now introduce shocks with different Mach numbers to hit the Rayleigh-Taylor interface at a fixed time $T = 1.85$, and observe the effects of shock waves when they interact with the
Figure 2.1: Rayleigh-Taylor flow at different time. Left (a): $T = 1.85$; middle (b): $T = 2.1$; right (c): $T = 2.5$. Density $\rho$, 45 equally spaced contour lines.
RTI flow.

First we put shock waves with Mach numbers 6, 12, 18, 24 into the computational domain from the top boundary at different times, so that different shock waves hit the head of Rayleigh-Taylor flow at the same time $T = 1.85$. The unshocked fluid is the top fluid state: $\rho = 1, p = 2.5, u = v = 0$. Since the moving shock waves make the RT interface move downward, in order to observe the development of the RT interface at a later time, we extend the computational domain in the simulations to avoid the RT interface moving out of the computational domain. For the shock waves hitting the head of the RT interface, we extend the computational domain in the $y$-direction to $[-20, 1]$. The initial condition for the domain $y < 0$ is the mean flow $\rho = 2, p = 1, u = v = 0$; the boundary condition at $y = -20$ is set to be an outflow. Figure 3.1 shows the Rayleigh-Taylor interface at four different times after a Mach 6 shock wave hits the head of RTI at $T = 1.85$ and passes through the RT interface. Cases for stronger shock waves are presented in Figures 3.2, 3.3 and 3.4, for shock Mach numbers 12, 18 and 24 respectively. We observe the effect of strong shock waves speeding up the transition of the RT flow to instability. The stronger the shock wave is, the earlier the RT flow develops into full instability.

The same conclusion is drawn when we put shock wave into the computational domain from the bottom boundary to hit the tail of the Rayleigh-Taylor interface. The unshocked fluid is the bottom fluid state: $\rho = 2, p = 1, u = v = 0$. Again, since the moving shock waves make the RT interface move upward, in order to observe the development of the RT interface at a later time, the computational domain in the $y$-direction is extended to $[0, 20]$. The initial condition for the domain $y > 1$ is the mean flow $\rho = 1, p = 2.5, u = v = 0$; the boundary condition at $y = 20$ is set to be an outflow. The right picture in Figure 3.5 shows the RT interface at $T = 3$ after a Mach 12 shock hits the tail of the RT interface at $T = 1.85$. Also in Figure 3.5, we compare the RT interface at $T = 6$ for the case without shock interaction, and the RTI flow at $T = 3$ for the two cases with shock interaction. The shock Mach number is 12 for both cases of the shock hitting the head and hitting the tail.
Figure 3.1: Mach 6 shock interacting with a Rayleigh-Taylor flow. Shock hits the head of the RT interface at $T = 1.85$. From left to right, the Rayleigh-Taylor interface at (a): $T = 1.9$; (b): $T = 2.3$; (c): $T = 2.6$; (d): $T = 3.0$. Density $\rho$; 45 equally spaced contour lines.

of the RT interface. We can observe that the shock waves do speed up the transition of the RT flow to full instability in each case.

4 Interaction of shock waves and the RTI

In this section, we take a more detailed inspection on the physical consequences of a shock wave interacting with an evolving RT flow. In particular, we are interested on whether the shock wave will diminish (as in the case of rotating flow) or enhance (as in the case of magnetic fluid) the growth of the mixing zone. Furthermore, we will investigate how such an effect depends on the strength of the shock.

Figures 3.1 — 3.4 show the time-dependence of the RTI flows after being hit by a shock wave at Mach number 6, 12, 18, and 24. As the strength of the shock wave increases, the
Figure 3.2: Mach 12 shock interacting with a Rayleigh-Taylor flow. Shock hits the head of the RT interface at $T = 1.85$. From left to right, the Rayleigh-Taylor interface at (a): $T = 1.9$; (b): $T = 2.3$; (c): $T = 2.4$; (d): $T = 2.6$. Density $\rho$; 45 equally spaced contour lines.
Figure 3.3: Mach 18 shock interacting with a Rayleigh-Taylor flow. Shock hits the head of the RT interface at $T = 1.85$. From left to right, the Rayleigh-Taylor interface at (a): $T = 1.9$; (b): $T = 2.2$; (c): $T = 2.3$; (d): $T = 2.4$. Density $\rho$; 45 equally spaced contour lines.
Figure 3.4: Mach 24 shock interacting with a Rayleigh-Taylor flow. Shock hits the head of the RT interface at $T = 1.85$. From left to right, the Rayleigh-Taylor interface at (a): $T = 1.9$; (b): $T = 2$; (c): $T = 2.1$; (d) $T = 2.2$. Density $\rho$; 45 equally spaced contour lines.
Figure 3.5: Left (a): the Rayleigh-Taylor flow at $T = 6.0$; middle (b): Mach 12 shock hitting the Rayleigh-Taylor interface from the top, $T = 3.0$; right (c): Mach 12 shock hitting the Rayleigh-Taylor interface from the bottom, $T = 3.0$. The time when shock hits the RT interface is $T = 1.85$ for both cases of the shock coming from the top and from the bottom. Density $\rho$; 45 equally spaced contour lines.
locations of the RTI interface move downward with the shock wave hitting the head of the interface.

We first look at the observation time at T=1.9 in Figures 3.1a, 3.2a, 3.3a, and 3.4a. Recall that the shock hits the evolving RTI flow at T=1.85, therefore, we are observing in these figures the effects of the shock strength at an early time. We can see a modest positive correlation between the strength of the shock wave and the growth of height and complexity of the structures (with the development of the associated secondary instability – Kelvin-Helmholtz Instability).

We now look at time T=2.3 for Mach 6, 12, and 18 (Figures 3.1b, 3.2b, and 3.3c). It is apparent that a strong shock will speed up the development of the mixing layer. With the Mach 6 shock wave, the two structures are still well organized. However, the symmetry is already lost when the Mach 12 shock is employed. When the Mach number reaches 18, we note that the width of the mixing zone has increased significantly. Also, the two symmetric structures in Figure 3.1b have already merged into one.

Comparing Figure 3.1c and Figure 3.2d (Mach 6 and 12 at T=2.6), we observe that the two structures are at relatively early and later stage of merging process, respectively. The similar trend is also observed in Figures 3.2c and 3.3d (at T=2.4). For the case of Mach 12, the structures are in the relatively late stage of the merging, but this process is already completed when the Mach number is increased to 18.

When the Mach numbers become very high at 18 and 24, we find that the structures of mixing layer are quite similar at T=2.2 (Figures 3.3b and 3.4d). We conjecture that after exceeding some large Mach number, the values of this important parameter may become irrelevant to the detailed process of disrupting the development of a RTI flow by a shock wave. Clearly, additional studies are needed.

Finally, we investigate whether the direction of the shock wave has any consequence on the future development of the mixing layer. Figures 3.5a-c suggest that there is a difference in the resulting mixing layer.
5 Concluding remarks

We observe, through a systematic numerical simulation of the two dimensional Navier-Stokes equation, that shocks interacting with a Rayleigh-Taylor interface can speed up the transition to full instability for the Rayleigh-Taylor interface significantly. Stronger shocks are more effective in this speed-up process. This conclusion is valid regardless of whether the shock hits the head or the tail of the Rayleigh-Taylor interface.

References


