Low-Redshift Cosmic Baryon Fluid on Large Scales: a Fully Developed Turbulence

Ping He\textsuperscript{1}, Long-Long Feng\textsuperscript{2}, Chi-Wang Shu\textsuperscript{3}, Li-Zhi Fang\textsuperscript{4}

\textsuperscript{1} Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China
\textsuperscript{2} Purple Mountain Observatory, Nanjing 210008, China
\textsuperscript{3} Division of Applied Mathematics, Brown University, Providence, RI 02912
\textsuperscript{4} Department of Physics, University of Arizona, Tucson, AZ 85721

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We investigate the statistical properties of cosmic baryon fluid in the nonlinear regime, which is crucial for understanding the large-scale structure formation of the universe. We show that the intermittency of the velocity field of cosmic baryon fluid at redshift \(z = 0\) in the scale range from the Jeans length \((\sim 0.1 - 0.3 \, \text{h}^{-1} \, \text{Mpc})\) to about \(10 \, \text{h}^{-1} \, \text{Mpc}\) can be extremely well described by the She-Lévêque’s universal scaling formula for fully developed turbulence. The baryon fluid also possesses the features: (1) the most singular dissipative structures are filaments, and (2) the relationship between the intensities of fluctuations is hierarchical. These results strongly indicate that the highly evolved cosmic baryon fluid is in the state as a fully developed turbulence.

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Introduction. The formation and evolution of large-scale structure of the universe is governed by the gravitational clustering of cosmic matter, in which about 72\% is dark energy, 24\% cold dark matter, and 4\% baryon. Most baryonic matter is in the form of gas. Therefore, the dynamic behavior of the density and velocity fluctuations of cosmic baryon fluid is dominated by the underlying gravitational potential of dark matter. In the linear regime, the baryon fluid follows the mass and velocity fields of collisionless dark matter point-by-point. In the non-linear regime, however, as first pointed out by Shandarin and Zeldovich in their early study of structure formation, the gravitational clustering of cosmic matter on large scales is similar to turbulence [1]. They emphasized that the motion of self-gravitation matter in the expanding universe is like that of noninteracting matter moving by inertia. In other words, cosmic matter underwent a scale-free evolution somewhat like fully developed turbulence in inertial range.

Nevertheless, the dynamical difference between the structure formation of cosmic matter and turbulent flow in incompressible fluid is obvious. The latter is rotational in general [2], while the former is irrotational, because vorticities do not grow during the clustering [3]. In turbulence, energy passes from large to the smallest eddies, and finally dissipates into thermal motion, while cosmic baryonic gas falls into massive halos to form structures, including light-emitting objects.

Yet, more and more similarities of the nonlinear behavior of cosmic baryon fluid to turbulence have been identified in recent years. For instance, the transmitted flux of quasar’s Ly\(\alpha\) absorption is found to be intermittent as turbulence [4]. The probability distribution functions (PDF) of the velocity differences across a distance \(r\), \(\delta v_x \equiv \left[ (v(x + r) - v(x)) / r \right]\), are scaling, when \(r\) is larger than the Jeans length [3]. Although cosmic baryon fluid is passive substance in comparing with dark matter, it decouples, in the non-linear evolutionary stage, from the underlying dark matter field [6], like the turbulence of passive substance developed in the underlying random field [7]. Moreover, the dynamical equation of cosmic baryon fluid of growth modes is reduced to be the stochastic force driven Burgers’ equation [8], which is well known in the theory of turbulence. Therefore, an important question is how well and why that the dynamical state of cosmic baryon fluid at low redshifts can be described as fully developed turbulence. In this Letter, we investigate this problem in the context of the universal scaling law of fully developed turbulence. We first show that the universal scaling does exist in the cosmic baryon fluid, and then analyze the underlying physical picture of this result.

Intermittent exponent. The universal property of fully developed turbulence is measured by the structure functions \(S_p(r)\) and intermittent exponent \(\zeta_p\), defined as

\[
S_p(r) = \left\langle |\delta v_x|^p \right\rangle \sim r^{\zeta_p}.
\]

Based on dimensional argument of hierarchical evolution, Kolmogorov in 1941 predicted that for fully developed turbulence on scales of inertial range, the intermittent exponent is \(\zeta_p = p/3\) [9]. Experimental and numerical results do not, however, support the \(p/3\) law for \(p > 3\). It should be attributed to intermittency, i.e. turbulence field in inertial range is characterized by stronger non-Gaussianity on smaller scales [10]. A remarkable development was made by She and Leveque [11] (SL henceafter). They proposed that the non-Gaussian behavior of fully developed turbulence is determined by the hierarchical structure originated from the Navier-Stokes equation, and then predicted that the \(p/3\) law should be replaced by

\[
\zeta_p / \zeta_3 = p / 9 + 2 \left[ 1 - \left( 2 / 3 \right)^{p/3} \right].
\]

This formula is in excellent agreement with experiments. Since this model does not contain adjustable parameters, the success of eq.(2) shows that the state of fully developed turbulence is universal. The SL formula actually is
based on the dimensional argument of hierarchical evolution, a generalized SL formula is [12]

\[
\zeta_p/\zeta_0 = [1 - C(1 - \beta^3)]p/3 + C(1 - \beta^p).
\]

Eq. (3) returns to eq. (2) if the parameters \( C = 2 \) and \( \beta^3 = 2/3 \). \( C \) is the Hausdorff codimension of the dissipative structures. We will discuss the details of the parameters \( C \) and \( \beta \) later. Eqs. (2) and (3) have been successfully justified in a variety of turbulence, including turbulence in astrophysical systems [12-14]. Namely, the SL scaling law of structure function is likely to be universal to characterize the fully developed turbulence.

To investigate cosmic baryon fluid, we use the cosmological hydrodynamic simulation samples produced by the code WIGEON (Weno for Intergalactic medium and Galaxy Evolution and formation) [15]. This is a hybrid hydrodynamic/N-body simulation, consisting of the WENO algorithm [16] for baryonic fluid, and N-body simulation for particles of dark matter. The baryon fluid obeys the Navier-Stokes equation, and is gravitationally coupled with collisionless dark matter. We use the \( \Lambda \)CDM cosmological model with parameters consistent with the recent observations of the cosmic microwave background radiation [17]. The linear power spectrum of mass density perturbations is taken from the fitting formulae of Eisenstein & Hu [18]. The atomic processes in the plasma of hydrogen and helium of primordial composition, including ionization, radiative cooling and heating, are modelled in the same way as in Cen [19].

The simulations were performed in a periodic, cubic box of size 25 h\(^{-1}\)Mpc with a 192\(^3\) grid and an equal number of dark matter particles. The simulations start at a redshift \( z = 49 \). A uniform UV-background of ionizing photons is switched on at \( z > 10 \) to heat the gas and reionize the universe. The temperature of the baryonic gas generally lies in the range \( 10^{4-6} \) K, and the speed of sound \( v_s \) in the baryonic gas is only a few km s\(^{-1}\) to a few tens km s\(^{-1}\). The Jeans length \( \lambda_J \) yields a term like the viscosity \( \mu \approx v_s \lambda_J \). On the other hand, the bulk velocity of the baryonic gas is of the order of hundreds km s\(^{-1}\) [20]. Therefore, the Reynolds number would be larger than \( \sim 100 \) if the scales under consideration are larger than the Jeans length \( \lambda_J \), which is in the range \( \sim 0.1 - 0.3 \) h\(^{-1}\) Mpc for redshifts \( z < 4 \) [21].

These samples are successful to model the statistical features of the observed transmitted flux of Ly\(\alpha\) [22]. Hence it would be suitable to study whether the universal scaling law eq. (2) is available for cosmic baryon fluid. For this purpose, we randomly sampled 500 one-dimensional subsamples, with each one containing 192 data points \( (500 \times 192 = 9.6 \times 10^4 \) total points.\) At each point, the peculiar velocity and mass density of the baryonic gas along this line is recorded.

We calculated the moments of velocity difference \( \delta v_p \) for \( r \) from 0.3 to 8 h\(^{-1}\) Mpc, of which the lower limit is given by the Jeans length, and the upper limit is given by the size of the simulation box. The intermittent exponent \( \zeta_p \) for the sample at redshift \( z = 0 \) is shown in Fig. 1.

The error bars are from the average over the 500 samples. The \( \zeta_p \) of SL formula [eq. (2)] is also shown in Fig. 1. It shows clearly that the intermittency of cosmic baryon fluid is remarkably consistent with the model eq. (2) on all orders from \( p = 1 \) to 15.

Our sample is very different from experimental and numerical samples [23] used to test the SL formula. Therefore, the success shown in Figure 1 further supports that the hypotheses used to derive the SL formula eq. (2) are universal so that it is also available for cosmic baryon fluid. In this sense, the non-Gaussian behavior of the cosmic baryon fluid in an expanding universe at redshift \( z = 0 \) can be essentially described as the statistically quasi-equilibrium state of fully developed turbulence.

**Singular dissipative structures.** Simply speaking, the underlying physical picture of the SL formula is as follows. In inertial range, the fluid evolution is governed by scale-covariant interactions from the Navier-Stokes equations. The kinetic energy of fluid is dissipated in singular structures, and the energy dissipation on different scales satisfies hierarchical relation. The parameter \( C \) is assumed to be the Hausdorff codimension of the most dissipative structures.

Although the clustering of baryon fluid is governed by the gravity of the background mass field, the expansion of the universe eliminates the gravity of the uniformly distributed matter. The peculiar motion of baryon fluid feels only the gravity given by the fluctuations of the randomly distributed dark matter. The evolution of the gravitational potential is very slow. It remains in almost linear regime even at low redshifts. Moreover, the random potential of gravity is scale-free, as the power spectrum of mass density perturbations of dark matter is Gaussian and scale-free in the scale range from \( \sim 10 \) h\(^{-1}\) Mpc to the Jeans length. Thus, in the nonlinear evolution of baryon fluid at low redshifts, the power of perturbations transfers from larger to smaller scales. Con-

![Fig. 1: Intermittent exponent \( \zeta_p \) of cosmic baryon fluid at redshift \( z = 0 \). The mean error bars are from the average over samples on scale range from \( r = 0.3 \) to 8 h\(^{-1}\) Mpc. The solid line is the She-Leveque formula eq. (2).](https://example.com/figure1.png)
considering the expansion of the universe is slow relative to the dissipation of the turbulence, and the baryon fluid should approximately achieve a statistically stationary state. Therefore, the evolution of cosmic baryon fluid on scales larger than the Jeans length is similar to that of turbulence in the inertial range.

In eq. (2) the most singular structures are assumed to be one-dimensional filaments, or \( C = 2 \). Thus, Figure 1 indicates that during the structure formation of cosmic baryon fluid, the most singular structures should be filaments. This result is consistent with the samples. Figure 2 shows a typical density contour of baryonic gas in a slice of 0.26 \( h^{-1} \) Mpc thickness of our samples at \( z = 0 \). One can clearly see the filament structures.

Hierarchical relation. The parameter \( \beta \) describes the intermittency of the hierarchical evolution. It has been well known that the non-linear clustering of large-scale structures of the universe is hierarchical. The \( N \)-point correlation function of mass density fluctuations is assumed to be expressed by the linked 2-point correlation function as \( \langle \delta \rho^N \rangle \propto \langle \delta \rho^2 \rangle^N \) where \( \delta \rho \) is the density fluctuation field of the cosmic matter on scale \( l \) [24]. In hydrodynamics, this relation is somewhat called pure hierarchical relation [25]. It has been shown, however, that this hierarchical relation can not be satisfied by the observed intermittency if the proportional coefficient between \( \langle \delta \rho^N \rangle \) and \( \langle \delta \rho^2 \rangle^N \) is scale-independent [26].

In the universal description of turbulence, the hierarchy is given by the following equation [12]

\[
F_{p+1}(r) = A_p F_p(r)^\beta F_\infty(r)^{1-\beta'},
\]

where \( F_p(r) \equiv S_{p+1}(r)/S_p(r) \), and number \( A_p \) are scale \( r \) independent. \( F_{p+1}(r) \) describes the intensity of fluctuations. The larger the \( p \) of \( F_p(r) \), the higher the intensity of fluctuations. The most intermittent structures are described by \( F_\infty(r) \). Eq. (4) gives the hierarchical relation between fluctuations with different intensities. Eq. (4) is an invariant relation between \( F_p(r) \) and \( F_{p+1}(r) \) with respect to a translation in \( p \). The parameter \( \beta' \) lies in the range \( 0 < \beta' < 1 \). It measures the degree of intermittency of turbulent flow. For \( \beta' = 1 \), the field is not intermittent. While for \( \beta' < 1 \), the smaller the \( \beta' \), the stronger the intermittency.

From eq. (4), we have

\[
\frac{F_{p+1}(r)}{F_2(r)} = A_p \left( \frac{F_p(r)}{F_1(r)} \right)^{\beta'}.
\]

If \( A_p \) is \( p \) independent, we should have

\[
\ln F_{p+1}(r)/F_2(r) = \beta' \ln F_p(r)/F_1(r) + \text{const.}
\]

From eq. (1) and (4), we have \( \beta'/\zeta_3 = \beta' \). Eq. (6) does not contain term \( F_\infty(r) \), and therefore can be directly used to test the assumption of hierarchy. Figure 3 presents the relation of \( \log F_{p+1}(r)/F_2(r) \) vs. \( \log F_p(r)/F_1(r) \), which is perfectly a straight line, with the slope being \( \beta' = 0.56 \). Moreover, from \( \zeta_3 = 1.55 \pm 0.42 \), we have \( (2/3)^{1/3} = 0.56 \pm 0.15 \). Therefore, the assumption of a constant \( A_p \) (\( p \)-independence) is tenable. The hierarchical relation eq. (4) with constant \( A_p \) and \( \beta' \) provides a good description of the hierarchical structures of cosmic baryon fluid.

Discussion and conclusion. We showed that all the moments of the fluctuations of velocity field of cosmic baryon fluid at \( z = 0 \) on scales larger than the Jeans length have a power-law dependence on scales, and the intermittent exponents obey the universal law, which depends only on (1) the dimension of the most singular dissipative structures and (2) the hierarchical relation between structures with various intensities of fluctuations. This result strongly indicates that the highly nonlinear evolution would lead to the cosmic baryon fluid reaching a statistically quasi-equilibrium state as fully developed.
turbulence. In view of this picture, we can say that in the highly nonlinear regime, the statistical properties, especially the intermittent behavior, of the velocity fluctuations are actually independent of the details of the cosmic baryon fluid, such as parameters related to the processes on scales less than the Jeans length. The state depends only on the dimension of dissipative structures and the exponent $\beta'$.

Since mass density field is related to the velocity field via continuity equation, the Navier-Stokes dynamics, or the Burgers equation, and the mass density field of cosmic baryon fluid should also be governed by the universal properties of the fully developed turbulence. Actually, Fig. 2 shows the filamentary structures of mass density perturbations. We believe, some statistical features, which have already been recognized in large-scale structures, would be directly in consequence of the universal properties of cosmic baryon fluid. In this context we may first address the log-normal distribution of cosmic baryon gas, which is in good agreement with observations of Lyα forests [27]. As has been shown, in the SL formula, the PDF of fluctuations is of log-Poisson [25].

It is also found recently that the non-Gaussian behavior of cosmic baryon fluid can be approximately described as a superposition of baryonic clouds [28]. We found that the moments of Fig. 1 can also be approximately fitted with the SL formula with $C \simeq 3$ and $\beta'^{2} > 2 / 3$. That is, if we try to approximately attribute the non-Gaussian behavior of baryon fluid to singular structures with dimension $\sim 0$, or $C \simeq 3$, the relation between fluctuations with different intensities is still given by eq.(4) with $\beta' < 1$, i.e. the clouds of baryon fluid are intermittent, and the clouds on different scales may not be simply self-similar.

The velocity and mass density field on scales larger than the Jeans length are the basic environment of the formation of the luminous objects. Therefore, the universal properties of the quasi-equilibrium state would play the role of the common initial condition for the formation of structures on small scales. We have also studied the intermittent properties of baryon fluid at some high redshifts. We found that at least at $z = 0.5$, the baryon fluid has already shown all the features of a fully developed turbulence. The details will be reported in a near future.

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